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Soliton Light Pulse Transport in a Quantum Wire of Nonlinear Ionic Crystal

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We discuss theoretically the transport of a light pulse along a quantum wire made of a nonlinear ionic crystal. Under the adiabatic approximation, the propagation of the axial component of the electric field along the quantum wire has soliton properties and its distribution in the cross section of the quantum wire still approximately satisfies the Bessel equation. The size effect of the quantum wire on the dispersion relation of the polaritons is also discussed.

KEY WORDS: quantum wire; polariton; soliton.

1. INTRODUCTION

In some ionic crystals, the nonlinear interaction between ions of a primitive cell is strong. The coupling between electromagnetic wave and optical phonon mode is then enhanced. The dispersion relation of the resulted polaritons involves not only frequency, but also field intensity. This new property of the dispersion relation generates many nonlinear optical effects. For example, the light pulse propagation in a bulk material of this kind of ionic crystal can form a soliton (Niu, 2002). In this paper, we discuss the dispersion relation of the nonlinear polaritons in a quantum wire made of this kind of ionic crystal and the propagation of a light pulse along the axis of the quantum wire, in order to find new characters different from that of bulk materials.

2. TRANSPORT OF A LIGHT PULSE ALONG A QUANTUM WIRE MADE OF NONLINEAR IONIC CRYSTAL

In Niu *et al.* (2001), we discussed polaritons resulted from the coupling between electromagnetic wave and optical phonon mode transporting in an isotropic

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ionic crystal. Considering the nonlinear interactions between ions, the nonlinear dielectric function of this ionic crystal can be written as

$$\epsilon(\omega) = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{\omega_{TO}^2 - \omega^2}\omega_{TO}^2 + 3\frac{b_{12}^4 g|E(\omega)|^2}{(\omega^2 - \omega_{TO}^2)^4 \epsilon_0}. \tag{1}$$

In this equation, $\epsilon_s, \epsilon_\infty$ are the static and the high frequency dielectric constant respectively, ω_{TO} is eigenfrequency of the transverse optical phonon mode, $E(\omega)$ is the electric field strength of the light for a given frequency in an isotropic ionic crystal, b_{12} is a phenomenal coefficient in Huang's equations (Huang, 1988) that describe the coupling between long wavelength optical phonon mode and electromagnetic wave, g is the nonlinear coefficient, c is light velocity in vacuum, and ϵ_0 is permittivity of vacuum. For brevity, Eq. (1) can be rewritten as

$$\epsilon(\omega) = \epsilon_1 + \epsilon_2|E(\omega)|^2, \tag{2}$$

where ϵ_1 is the linear dielectric constant and ϵ_2 represents the nonlinear effect. For a quantum wire, optical phonon modes include bulk-like modes and surface modes due to quantum size effect (Xia, 1995). If the value of ω_{TO} in Eqs. (1) and (2) adopts that of bulk-like modes, Eqs. (1) and (2) are still valid for a quantum wire.

We discuss a cylindrical quantum wire in this paper, which is surrounded by a metal material. The radius of the wire is R. Suppose the electromagnetic wave pulse propagates along the axial, its electric field strength $\mathbf{E}(\rho, \varphi, z, t)$ can be expressed in the cylindrical coordinate as

$$\mathbf{E}(\rho, \varphi, z, t) = \mathbf{X}(\rho, \varphi) Y(z, t) e^{i(k(\omega_0)z - \omega_0 t)}, \tag{3}$$

where $\mathbf{X}(\rho, \varphi)$ is the time independent solution of the electric field strength in the cross-section and $Y(z, t)$ is an envelope function of the electromagnetic wave pulse along the axis. The wave number k should be real, or $\mathbf{E}$ and $\mathbf{H}$ will attenuate along the axial and will not be a propagating wave. ω_0 is the center frequency of the pulse. If there is no free charge in the wire, the electromagnetic wave transport equation can be obtained from the Maxwell equations:

$$\Delta \mathbf{E} - \mu_0 \ddot{\mathbf{D}} = 0 \tag{4}$$

This is a vector wave equation. For a TM electromagnetic wave (i.e., $H_z = 0$), substituting Eqs. (2) and (3) into Eq. (4), we can obtain that the z component of the electric field satisfies the following equations in the cylindrical coordinate:

$$\Delta_2 XY\, e^{i(k(\omega_0)z - \omega_0 t)} + X(Y^N - 2ik(\omega_0)Y' - k(\omega_0)^2 Y)\, e^{i(k(\omega_0)z - \omega_0 t)} - \omega_0 \ddot{D} = 0 \tag{5}$$

Here $\Delta_z = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho}) + \frac{1}{\rho^2}\frac{\sigma^2}{\partial\varphi^2}$ and Y' is given by $\frac{\partial Y}{\partial z}$. The relation between the Fourier amplitude of the dielectric displacement vector $D(\omega)$ and that of the electric field $E(\omega)$ is

$$D(\omega) = \epsilon_0 \epsilon(\omega) E(\omega). \tag{6}$$

Using the Fourier transform, $\ddot{D}(t)$ in Eq. (6) can be written as

$$\begin{aligned}
\ddot{D}(t) &= -\frac{1}{2\pi}\int_{-\infty}^{\infty} \omega^2 D(\omega)\, e^{i\omega t} d\omega \\
&= -\frac{\epsilon_0}{2\pi}\int_{-\infty}^{\infty} \omega^2 \epsilon(\omega)\, e^{i\omega t} d\omega \int_{-\infty}^{\infty} E(t')\, e^{-i\omega t'} dt' \\
&= -\frac{\epsilon_0}{2\pi}\int_{-\infty}^{\infty} \omega^2 \epsilon(\omega) e^{i\omega t} d\omega \int_{-\infty}^{\infty} XY(t')\, e^{-i(\omega-\omega_0)t' - ik(\omega_0)z} dt' \\
&= -\frac{\epsilon_0 X}{2\pi}\, e^{i\omega_0 t - ik(\omega_0)z} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \omega^2 \epsilon(\omega)\, e^{i(\omega-\omega_0)(t-t')} Y(t') d\omega dt'
\end{aligned} \tag{7}$$

Considering the dispersion of the medium, we can expand $\omega^2\epsilon(\omega)$ in the preceding equation around ω_0 as

$$\omega^2\epsilon(\omega) \approx \omega_0^2\epsilon(\omega_0) + \omega_0^2\epsilon_2\langle E^2\rangle + [\omega^2\epsilon(\omega)]'(\omega-\omega_0) + \frac{1}{2}[\omega^2\epsilon(\omega)]''(\omega-\omega_0)^2, \tag{8}$$

where the values of the derivatives should be taken at $\omega = \omega_0$ and ϵ_2 represents the strength of the nonlinearity. Equation (7) can be rewritten as

$$\begin{aligned}
\ddot{D}(t) = -\epsilon_0 X\, e^{i(k(\omega_0)z - \omega_0 t)} \Big\{ &\omega_0^2\epsilon(\omega_0)Y + \omega_0^2\epsilon_2\langle E^2\rangle Y \\
&- i[\omega^2\epsilon(\omega)]'\dot{Y} - \frac{1}{2}[\omega^2\epsilon(\omega)]''\ddot{Y} \Big\}.
\end{aligned} \tag{9}$$

Substituting Eq. (9) into Eq. (5) and dividing both sides by $XY\, e^{i(k(\omega_0)z-\omega_0 t)}$, we have

$$\begin{aligned}
&\Delta_2 X/X + (Y'' - 2ik(\omega_0)Y')/Y - k(\omega_0)^2 + \frac{1}{c^2}\omega_0^2\epsilon(\omega_0) + \frac{1}{c^2}\omega_0^2\epsilon_2\langle E^2\rangle \\
&\quad + \frac{1}{c^2}\left\{ i[\omega^2\epsilon(\omega)]'\dot{Y} - \frac{1}{2}[\omega^2\epsilon(\omega)]''\ddot{Y} \right\}/Y = 0.
\end{aligned} \tag{10}$$

Only the term ϵ_2 in this equation contains the product of X and Y. If the nonlinearity is not very strong, we can use the adiabatic approximation and the mean field method (i. e., replacing X^2 in $\langle E^2\rangle$ with the average of it on the cross section of the quantum wire). Thus Eq. (10) can be resolved into the following two equations.

$$\Delta_2 X + \left[\frac{\omega_0^2 \varepsilon(\omega_0)}{c^2} - k^2(\omega_0)\right] X = 0, \tag{11}$$

$$Y'' - 2ik(\omega_0)Y' + \frac{1}{c^2}\left\{\omega_0^2 \epsilon_2 \langle E^2\rangle Y - i[\omega^2\epsilon(\omega)]'\dot{Y} - \frac{1}{2}[\omega^2\epsilon(\omega)]''\ddot{Y}\right\} = 0. \tag{12}$$

From these two equations, we can derive the static solution of the z component of the electric field of the light pulse in the cross-section of the quantum wire and its propagation properties along the z axis, respectively.

3. DISPERSION RELATION OF THE POLARITONS AND SOLITON PROPERTIES OF THE PROPAGATION OF LIGHT PULSE ALONG THE AXIS

Equation (11) is a Helmholtz equation of which the solution is

$$X = C J_m(\beta\rho) \begin{Bmatrix} \cos m\varphi \\ \sin m\varphi \end{Bmatrix}, \tag{13}$$

here $\beta = \frac{\omega_0^2 \epsilon(\omega_0)}{c^2} - k^2(\omega_0)$. C is a constant that determines the strength of the electric field and $J_m(x)$ is a Bessel function of mth order. The boundary conditions for electric field of the electromagnetic wave make the following result.

$$\beta R = x_n^{(m)}, \tag{14}$$

where x_n^m is the nth zero of $J_m(x)$. From this equation we can get the dispersion relation of polaritons in this quantum wire made of a nonlinear ionic crystal.

$$k(\omega_0)^2 = \frac{\epsilon_1(\omega_0)\omega_0^2}{c^2} - \left(\frac{x_n^{(m)}}{R}\right)^2. \tag{15}$$

We can see that the dispersion relation of polaritons of a quantum wire differs from that of a bulk material due to quantum size effect. The dispersions of k and k' are shown in Fig. 1 and Fig. 2. We can see that k' is always positive for different radii. At low frequencies, k'' is negative for small radii and turns into positive for large radii. For very large radii, the properties of k'' are the same as that of a bulk material. At frequencies near ω_{LO}, the smaller the radius is, the faster k'' increases.

Using slowly varying envelope approximation, we can neglect Y'' in Eq. (12). Since $\langle E^2\rangle = \frac{1}{2}|E_z|^2 = \frac{1}{2}|XY|^2$, Eq. (12) can be changed into

$$Y' + \frac{1}{v_g}\dot{Y} - i\frac{B_1}{2}\ddot{Y} + iB_2|Y|^2 Y = 0. \tag{16}$$

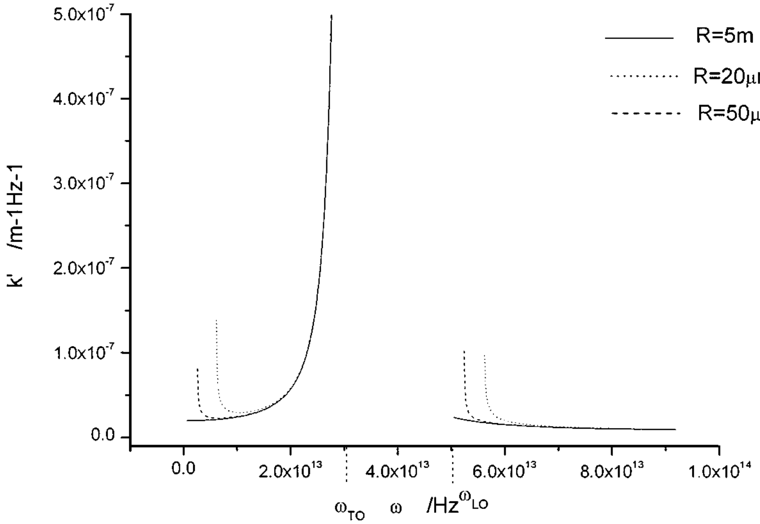

Fig. 1.

Here $v_g = \frac{2k(\omega_0)c^2}{[\omega^2\epsilon(\omega)]'}$ is the group velocity of the light pulse, $B_1 = \frac{[\omega^2\epsilon(\omega)]''}{2k(\omega_0)c^2}$ represents the dispersion of the group velocity, and $B_2 = \frac{\omega_0^2\epsilon_2}{4k(\omega_0)c^2}|X|^2$. Using the following transformation,

$$\left.\begin{aligned} \xi &= z, \\ \tau &= \tfrac{1}{\sqrt{-B_1}}\left(t - \tfrac{z}{v_g}\right), \\ u &= \sqrt{B_2}Y. \end{aligned}\right\} \tag{17}$$

Equation (16) can be transformed into the normalized nonlinear Schödinger equation (NLS)

$$i\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + |u|^2 = 0. \tag{18}$$

For initial condition

$$u(\tau, \xi = 0) = A\,\mathrm{sech}(\tau), \tag{19}$$

at $A = 1$, the following soliton solution of Eq. (18) can be obtained using the inverse scattering method (Li, 1992].

$$u(\tau, \xi) = \mathrm{sech}(\tau),\, e^{(i\xi/2)}. \tag{20}$$

This is the basic soliton solution of Eq. (18). For $A = N \geq 2$, N is a positive integer, the solutions to Eq. (18) are high order solitons. From the dispersion

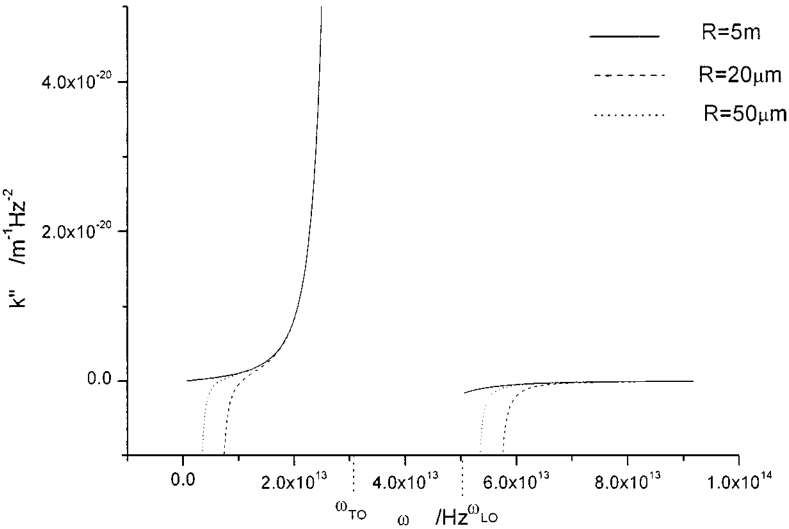

Fig. 2.

relation of polaritons (Eq. (15)) and Fig. 2, we can see that at low frequencies for small radii and frequencies larger than ω_{LO}, k'' is negative, which means that the group velocity dispersion is negative. Therefore at these frequencies, bright solitons can be realized. Dark solitons can be realized at frequencies lower than ω_{TO}, except those low frequencies at which k'' is negative.

4. CONCLUSIONS AND DISCUSSIONS

We discussed theoretically the transport properties of a light pulse along a quantum wire made of an ionic crystal with the consideration of nonlinear effects. Under the adiabatic approximation, the distribution of the axial component of the electric field still approximately satisfies the Bessel's function in the cross-section, and its propagation along the quantum wire has soliton properties. If a quantum wire made of an ionic crystal with strong nonlinear effects can be fabricated, the soliton properties of the transport of a light pulse will be of practical use and worth of attention. It should be noticed that the polaritons discussed in this paper is the coupled mode of an electromagnetic wave and a bulk-like phonon mode. There are also surface optical phonon modes in a quantum wire made of an ionic crystal. Polaritons resulted from the coupling between electromagnetic wave and surface optical phonon mode, its dispersion relation and the light pulse transport properties are not involved in this paper, and will be discussed in our future work.

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